# SELF-CONSISTENT MULTI-MODE TIME DOMAIN ANALYSIS OF GYROTRONS

A. Jöstingmeier[1], C. Rieckmann[1], and A. S. Omar[2]

[1] Technische Universität Braunschweig
Institut für Hochfrequenztechnik
Schleinitzstr. 23, D-38023 Braunschweig, Germany

[2] Technische Universität Hamburg-Harburg
Arbeitsbereich Hochfrequenztechnik
Wallgraben 55, D-21071 Hamburg, Germany

## ABSTRACT

A self-consistent time domain analysis of gyrotrons is presented which allows studying multi-mode, multi-frequency operation. The electromagnetic field in the gyrotron cavity is expanded with respect to complete sets of eigenfunctions so that space charge effects are included in the analysis. It is demonstrated that the strong numerical requirements of this method can be met by using a vector computer. The simulations show that the assumption of a monofrequent steady state operation of gyrotrons, which is made by the commonly used frequency domain methods is not always fulfilled. For a low Q gyrotron, both oscillation build-up and steady state operation is investigated including mode competition and window reflections.

## STATEMENT OF THE PROBLEM

In the last two decades gyrotrons have attracted a lot of attention as high-power millimeter-wave sources for fusion experiments ([1]–[7]). In 1982, Fliflet and some co-workers achieved an essential advance in gyrotron theory developing a self-consistent model which takes the effect of the electron beam on the electromagnetic field in the interaction resonator into account [5]. In the following years, this model has been refined ([6]–[7]) and has become a familiar tool for theoretical investigation and design of gyrotrons.

All of the models presented in [5]–[7] are based on frequency domain analysis since they assume that gyrotrons operate at a single-frequency in the steady state. On the other hand, gyrotron interaction cavities are strongly oversized due to wall losses which may give rise to multi-mode, multi-frequency operation. A self-consistent time domain model is consequently needed which takes both effects into account.

To illustrate the above statement a case in which multi-mode, multi-frequency operation is expected should be considered. For this purpose we discuss the effect of window reflections on gyrotron performance. Assuming a single-frequency steady state operation, reflections from the window can completely be avoided if the window is properly designed (half-wave transformer). Consequently, such a window does not affect the results in a frequency domain analysis.

On the other hand, gyrotrons have an output spectrum with a non-vanishing bandwidth. Keeping in mind that with half-wave transformers reflections can effectively be suppressed in a narrow band around the design frequency only, the window gives rise to significant reflections for those frequencies which considerably deviate from the design frequency. These reflections may lead to injection-locking of another mode than the design mode. Such an effect completely changes gyrotron performance with respect to both operating mode and operating frequency (output spectrum). Therefore, if the effect of window reflections is investigated, multi-mode, multi-frequency operation is likely. An analysis technique is hence required which allows taking these phenomena into account.

## DESCRIPTION OF THE MODEL

The trajectories of electrons through a gyrotron cavity may be described by two sets of differential equations: Maxwell's equations on one hand, with the electron charge density and current density as source terms, and the equations of motion for each electron on the other hand, with the electromagnetic field as an external force. The suggested method calculates both, the electromagnetic field and electron motion in the gyrotron resonator as they develop in time.

The computation of the electromagnetic field in the interaction region is based on the following technique [8]: By application of the equivalence principle, the apertures of a gyrotron resonator are short-circuited and the nonvanishing tangential electric field there is replaced by two surface magnetic currents at both sides of the short circuit, which are equal in magnitude and opposite in direction. The electromagnetic field inside the cavity is then expanded with respect to the solenoidal *and* irrotational eigenfunctions of the corresponding completely shielded resonator. In [9], it has been demonstrated that these eigenfunctions are complete and orthogonal so that space charge effects are included in the analysis.

Only for a few cavities the eigenfunctions can be derived analytically. The generalized scattering matrix method which is usually used for the computation of the resonance modes of microwave cavities is also applied to the computation of the irrotational eigenfunctions. A computer code for the calculation of both resonance modes and irrotational eigenfunctions of circularly symmetrical cavities has been developed. This code can actually handle cavities consisting of cascaded line sections, step discontinuities and tapers of circular waveguides.

In the modal expansion of the electromagnetic field, a harmonic time dependence is inherently assumed. A discrete Fourier transform is then necessary for the application of this procedure in time domain. Thus, at each step in time the time-dependent quantities which describe the excitation of the resonator field





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due to the electron beam are numerically transformed into a corresponding spectrum in frequency domain. Then, the field problem is individually solved for each spectral line within an appropriate band around the cyclotron frequency. Finally, the time-dependent electromagnetic field is obtained by superimposing the corresponding spectral terms.

A set of relativistic single-particle equations is used to describe the electrons motion. Following the analysis presented in [5], which is based on the fact that due to the strong uniform magnetic field in the axial direction the RF electromagnetic field gives rise to small perturbations for the helical trajectories of the electrons, new slow-time scale variables are introduced. Consequently, a single cyclotron harmonic is taken into account for the calculation of the electron-field interaction. This technique leads to a significant increase in stepsize for the numerical solution of the initial-value problem.

At each step in time, a bunch of electrons with uniformly distributed initial phases is injected into the cavity at the guiding centre radius. It is assumed that all electrons have the same transverse velocity so that effects due to spreads in the spatial and velocity distribution are neglected. On the other hand, space charge effects are automatically included in the analysis due to the completeness of the eigenfunctions which are used for the modal expansion of the electromagnetic field inside the cavity.

## NUMERICAL REQUIREMENTS

Compared to frequency domain methods ([5]–[7]), computational requirements of the proposed method are quite stringent. To illustrate this, some typical figures should be given: The electromagnetic field inside the gyrotron resonator has been expanded with respect to some ten eigenfunctions. Typically, 4096 spectral terms are considered for the Fourier transform of the time-dependent quantities which describe the excitation of the cavity eigenfunctions in a band from DC to twice the cyclotron frequency. Since the resonator equations are very frequency selective, it is sufficient to take a band of ±25% of the cyclotron frequency into account for each cavity eigenfunction which means that the field problem has to be solved 1024 times at each step in time. Due to the slow-time scale variables which are used in the equations of motion for the electrons and in the term describing the beam-field interaction, a stepsize of half a cyclotron period is permissible even if high accuracy is required and steady state operation is reached after some hundreds of cyclotron periods. Usually, a bunch of 16 electrons with uniformly distributed initial phases is injected into the gyrotron cavity at each step of time. For the structure investigated in [5], this means that the gyrotron cavity contains about 500 particles during steady state operation leading to a correspondingly large system of equations, which would have been hardly solvable as the self-consistent frequency domain model was developed a decade ago [5]. Nowadays, powerful vector computers are available which enables dealing with such large systems of equations. The presented method has been implemented on a medium-size machine with a peak performance of about 100 MFlops. Typically 5 hours cpu time are required to simulate oscillation build-up until steady state operation is reached.

## NUMERICAL RESULTS

Simulations for a wide range of operating parameters have been carried out for the 35 GHz $TE_{011}$ mode gyrotron, which has already been extensively investigated in [5] by the use of a self-consistent frequency domain analysis.

The gyrotron efficiency $\eta$ defined as the ratio of RF power to DC input power is shown in Fig. 1 as a function of time ($T_c$ denotes a cyclotron period.) during oscillation build-up. In a gyrotron, oscillations start from preoscillation noise which is not taken into account in the presented method. Instead of this, a small amount of energy is assumed to be initially present in the gyrotron cavity to start oscillations. Thus starting time depends on the initial conditions. On the other hand, the principal transient behaviour and the steady state operation are independent of the initial conditions. In Fig. 1, steady state operation is reached after about 500 cyclotron periods.

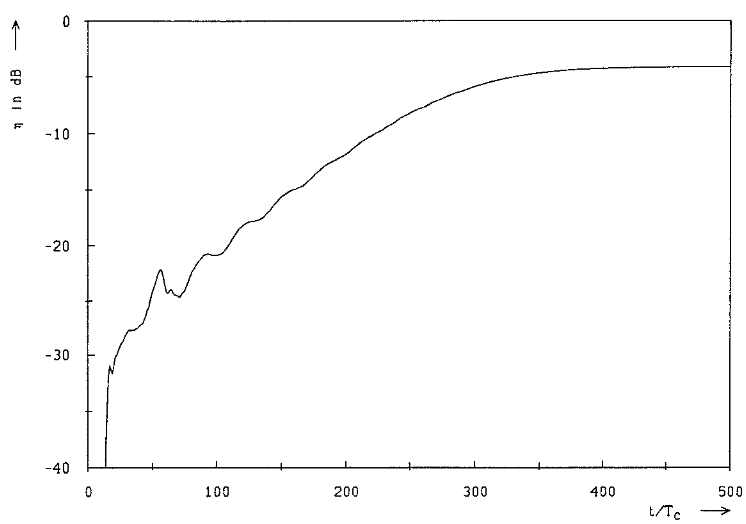


Figure 1: Efficiency as a function of time during oscillation build-up.

Fig. 2 summarizes the effect of varying the axial static magnetic field $B_{z0}$ on the spectral distribution of the expansion coefficient corresponding to the prevailing mode. In Fig. 2, $B_{z0}$ is uniformly increased in steps of 0.04 T from 1.32 T to 1.52 T. From the corresponding spectra it can be concluded that the assumption of a single-mode, monofrequent steady state operation is fulfilled in the range from $B_{z0} = 1.32$ T to $B_{z0} = 1.36$ T only. On the other hand, for higher values of the axial static magnetic field the complete spectral distribution of the cavity eigenfunction has to be taken into account in a certain band around the cyclotron frequency.

A comparison between numerical results from the present method and those corresponding to a modified self-consistent frequency domain model which takes the space charge effects into account is made. In Fig. 3, the comparison of $\eta$ as a function of $B_{z0}$ corresponding to our method (—) and the modified self-consistent frequency domain model (- -) (explicitly calculated points are marked by ∘ and △, respectively) is shown. In the range where both methods yield a solution, the agreement of the curves is excellent. On the other hand, the frequency domain analysis fails for $B_{z0} > 1.38$ T where the spectral distribution of the electromagnetic field becomes important. In this range of $B_{z0}$, the curve corresponding to the presented time domain method indicates the steady change from the $TE_{011}$ mode-operation to the $TE_{012}$ one. In a small range below $B_{z0} = 1.32$ T, the frequency domain model still predicts steady state operation of the gyrotron with good efficiency whereas in our method the beam current of 15 A is too small to start oscillations. This pheno-

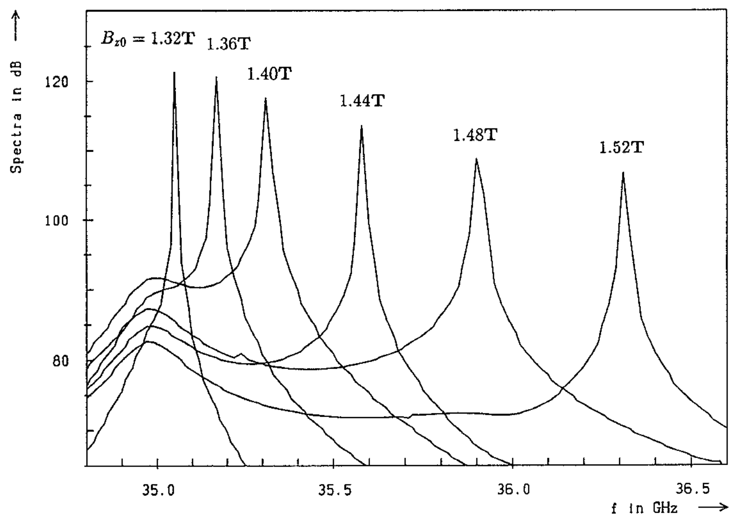


Figure 2: Spectra of the cavity expansion function corresponding to the mode dominating gyrotron operation with the axial static magnetic field as parameter.

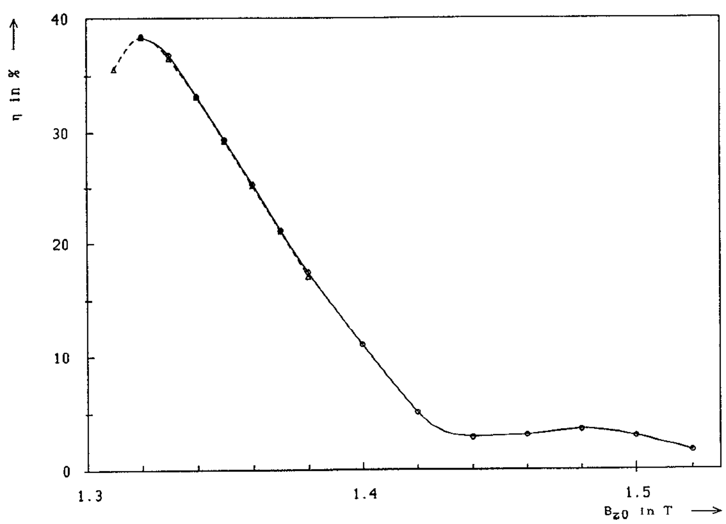


Figure 3: Comparison between time (—) and frequency (- -) domain analysis for the efficiency as a function of the axial static magnetic field.

menon may be explained by the fact that the frequency domain method is based on a search procedure starting from an initial guess of the unknown parameters. Consequently, steady state solutions may be found which cannot be reached by a simple start-up process beginning from "zero" (hard excitation).

Window reflections represent an example of great practical importance where strong multi-frequency operation is expected due to the external resonances between the interaction cavity and the window. In Fig. 4, the steady state spectra of some cavity expansion functions are presented for a 2λ-window in a distance of 50 cm from the gyrotron cavity. The external resonances which are characterized by an approximately constant spectral distance of about 200 MHz are present in all curves.

## CONCLUSIONS

A self-consistent time domain method for the analysis of transient behaviour and steady state operation of gyrotrons has been suggested. It has been shown that vector computers are suitable for a numerically efficient implementation of the proposed method. Parameter studies have been carried out for a low Q gyrotron. The effect of mode competition and window reflections on gyrotron performance has been studied. In some cases, multi-frequency operation has been observed for which none of the well-known frequency domain methods can be applied.

## ACKNOWLEDGEMENT

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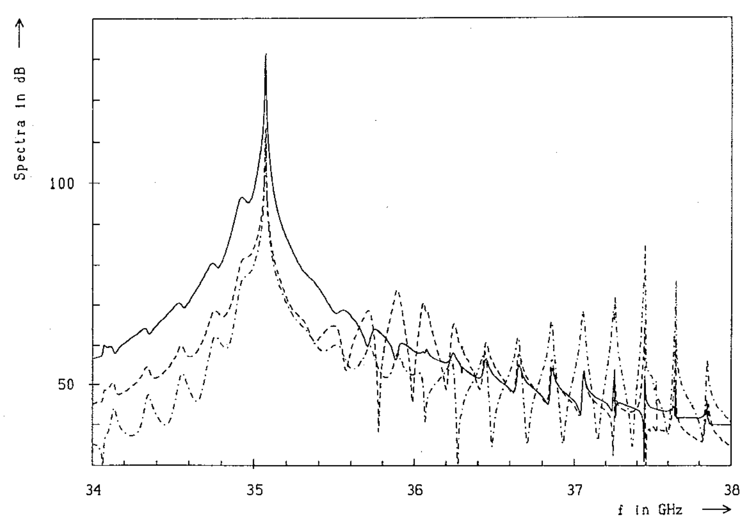


Figure 4: Effect of window reflections on gyrotron performance for a 2λ-window located 50 cm from the interaction cavity.